Application of Input-Output Analysis in Trade

Dr. Anusree Paul
Associate Professor
Indian School of Business and Finance
New Delhi

p_anusri@yahoo.com
Outline of the lecture:

• Background: Leontief Paradox: Input-Output Model
• Structure of the IO model
• Use of IO model in Trade – Vertical Specialisation (Hummel et. Al., 2001)
  - Input-output and national accounts
  - Input-output table and national accounts in international economy.
• WIOD and TiVA databases
• Domestic value addition and Job creation by exports
• Calculations of DVA and FVA of Export of India in MATLAB using WIOD database
• Relationship between SAMs and the National Accounts
• Comparison of characteristics of IO, SAM and CGE models
Background: Leontief Paradox

- An empirical test of the H-O model was carried out by Wassily Leontief using his invention called the input-output table for the United States, classified as the most capital abundant nation in all over the world. Therefore, one would expect that the US exports are more capital intensive relative to the US imports. However, Leontief’s results showed that the US seemed to export labour intensive goods and import capital intensive ones.
Background: Leontief Input-Output Model

• Professor Wassily Leontief deals with this particular question: “what level of output should each of the in n industries in an economy produce, in order that it will just be sufficient to satisfy the total demand for that product?”

• I-O model: inter-industry/inter-sectoral dependence.

• The rationale for the term input-output analysis: The output of any industry is needed as an input in many other industries, or even for that industry itself; therefore the "correct" level of output of a particular industry will depend on the input requirements of all the n industries.
Structure of an Input-Output Model

• **Assumptions:**

(1) each industry produces only one homogeneous commodity

(2) each industry uses a fixed input ratio (or factor combination) for the production of its output; and

(3) production in every industry is subject to constant returns to scale

• **Input – Output coefficient matrix, \( A = [a_{ij}] \):**
• The open model:

\[
\begin{bmatrix}
(1-a_{11}) & -a_{12} & \cdots & -a_{1n} \\
-a_{21} & (1-a_{22}) & \cdots & -a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
-a_{n1} & -a_{n2} & \cdots & (1-a_{nn})
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix}
=
\begin{bmatrix}
d_1 \\
d_2 \\
\vdots \\
d_n
\end{bmatrix}
\]

Or, \((I-A)X=d\)

- \(X\) = variable vector
- \(d\) = final demand (constant term) vector
- \((I-A)\) = Leontief matrix

If \((I-A)\) is non-singular, we can obtain its inverse and the unique solution is: \(X^* = (I-A)^{-1}d\)
Example:

Let us consider the 3-sector model: steel (s), food (f) and labour (l).

Consider:

- 1 new unit of steel (s) requires
  - 0.4 units of existing steel
  - 0.5 units of labour
- 1 new unit of food (f) requires
  - 0.1 units of food
  - 0.7 units of labour
- 1 new unit of labour (l) requires
  - 0.8 units of food
  - 0.1 units of labour
  - 0.1 units of steel
In matrix notation, we can write the *input coefficient matrix* (3 x 3) as:

\[ A = \begin{bmatrix}
0.4 & 0.0 & 0.1 \\
0.0 & 0.1 & 0.8 \\
0.5 & 0.7 & 0.1 \\
\end{bmatrix} \]

we further need a *production matrix* (3 x 1):

\[ X = \begin{bmatrix}
S \\
F \\
L \\
\end{bmatrix} \]

i.e., how much each sector is producing.

\[ AX = \begin{bmatrix}
0.4 & 0.0 & 0.1 \\
0.0 & 0.1 & 0.8 \\
0.5 & 0.7 & 0.1 \\
\end{bmatrix} \begin{bmatrix}
S \\
F \\
L \\
\end{bmatrix} \]

is the amount consumed by the economy.
\[
\begin{bmatrix}
0.4s + 0.1l \\
0.1f + 0.8l \\
0.5s + 0.7f + 0.01l
\end{bmatrix}
\]

Therefore, net production = \(X-AX\) = left over amount to satisfy the overall demand (domestic + export) = \(d\)

\[X-AX = d\]

Or, \((I-A)X=d\)

Or, \(X^* = (I-A)^{-1}d\)
Use of IO model in Trade - Measuring “Trade in Value-Added”

What is Trade in Value-Added?

Country A exports $100 of goods, produced entirely within A, to country B that further processes them before exporting them to C where they are consumed. B adds value of $10 to the goods and so exports $110 to C. Conventional measures of trade show total global exports and imports of $210 but only $110 of value-added has been generated in their production.
Vertical Specialisation/value added trade - Hummels et al. (2001)

Fig. 1. Vertical specialization.
Input-output and national accounts (WIOD structure)

<table>
<thead>
<tr>
<th>j</th>
<th>i</th>
<th>Ind.1</th>
<th>Ind.2</th>
<th>......</th>
<th>Ind. N</th>
<th>Final Consumption</th>
<th>Export</th>
<th>GO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$I^d_{11}$</td>
<td>$I^d_{12}$</td>
<td>......</td>
<td>$I^d_{nn}$</td>
<td>$C^d_1$</td>
<td>$E^d_1$</td>
<td>$y_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I^d_{21}$</td>
<td>$I^d_{22}$</td>
<td>......</td>
<td>$I^d_{2n}$</td>
<td>$C^d_2$</td>
<td>$E^d_2$</td>
<td>$y_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>Ind. N</td>
<td>$I^d_{n1}$</td>
<td>$I^d_{n2}$</td>
<td>......</td>
<td>$I^d_{nn}$</td>
<td>$C^d_n$</td>
<td>$E^d_n$</td>
<td>$y_n$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I^m_{11}$</td>
<td>$I^m_{12}$</td>
<td>......</td>
<td>$I^m_{nn}$</td>
<td>$C^m_1$</td>
<td>$E^{re}_1$</td>
<td>$M_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I^m_{21}$</td>
<td>$I^m_{22}$</td>
<td>......</td>
<td>$I^m_{2n}$</td>
<td>$C^m_2$</td>
<td>$E^{re}_2$</td>
<td>$M_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td></td>
<td>Ind. N</td>
<td>$I^m_{n1}$</td>
<td>$I^m_{n2}$</td>
<td>......</td>
<td>$I^m_{nn}$</td>
<td>$C^m_n$</td>
<td>$E^{re}_n$</td>
<td>$M_n$</td>
</tr>
<tr>
<td></td>
<td>VA (Value added)</td>
<td>$V_1$</td>
<td>$V_2$</td>
<td>......</td>
<td>$V_n$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>GO (Total output)</td>
<td>$y_1$</td>
<td>$y_2$</td>
<td>......</td>
<td>$y_n$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
i. Total domestic supply equals to use of domestic production for each product (in matrix notation):

\[
\begin{pmatrix}
I_{11}^d & \cdots & I_{1n}^d \\
\vdots & \ddots & \vdots \\
I_{n1}^d & \cdots & I_{nn}^d
\end{pmatrix}
+ \begin{pmatrix}
C_1^d \\
\vdots \\
C_n^d
\end{pmatrix}
+ \begin{pmatrix}
E_1^d \\
\vdots \\
E_n^d
\end{pmatrix}
\equiv \begin{pmatrix}
y_1 \\
\vdots \\
y_n
\end{pmatrix}
\]

Or,

\[
\begin{pmatrix}
a_{11} & \cdots & a_{1n} \\
\vdots & \ddots & \vdots \\
a_{n1} & \cdots & a_{nn}
\end{pmatrix}
\begin{pmatrix}
y_1 \\
\vdots \\
y_n
\end{pmatrix}
+ \begin{pmatrix}
C_1^d \\
\vdots \\
C_n^d
\end{pmatrix}
+ \begin{pmatrix}
E_1^d \\
\vdots \\
E_n^d
\end{pmatrix}
\equiv \begin{pmatrix}
y_1 \\
\vdots \\
y_n
\end{pmatrix}
\]

Or,

\[(1) \quad Y \equiv A_d \cdot Y + Y_d \quad \text{with} \quad Y_d = C_d + E_d\]
ii. Total imports equal to total use of imported products, i.e.,

\[
\begin{pmatrix}
   m_{11} & \cdots & m_{1n} \\ 
   \vdots & \ddots & \vdots \\ 
   m_{n1} & \cdots & m_{nn}
\end{pmatrix}
\begin{pmatrix}
   y_1 \\ 
   \vdots \\ 
   y_n
\end{pmatrix} + 
\begin{pmatrix}
   C_1^m \\ 
   \vdots \\ 
   C_n^m
\end{pmatrix} + 
\begin{pmatrix}
   E_1^{re} \\ 
   \vdots \\ 
   E_n^{re}
\end{pmatrix} \equiv 
\begin{pmatrix}
   M_1 \\ 
   \vdots \\ 
   M_n
\end{pmatrix}
\]

Or,

\[M \equiv A_m \cdot Y + Y_m\]
where
\[Y_m = C_m + E_{re}\]

iii. Finally, the supply table gives,

\[
\begin{pmatrix}
   1 & \cdots & 1 \\ 
   \vdots & \ddots & \vdots \\ 
   1 & \cdots & 1
\end{pmatrix}
\begin{pmatrix}
   a_{11} & \cdots & a_{1n} \\ 
   \vdots & \ddots & \vdots \\ 
   a_{n1} & \cdots & a_{nn}
\end{pmatrix} + 
\begin{pmatrix}
   1 & \cdots & 1 \\ 
   \vdots & \ddots & \vdots \\ 
   1 & \cdots & 1
\end{pmatrix}
\begin{pmatrix}
   m_{11} & \cdots & m_{1n} \\ 
   \vdots & \ddots & \vdots \\ 
   m_{n1} & \cdots & m_{nn}
\end{pmatrix} + 
\begin{pmatrix}
   \nu_1 \\ 
   \vdots \\ 
   \nu_n
\end{pmatrix} \equiv 
\begin{pmatrix}
   1 \\ 
   \vdots \\ 
   1
\end{pmatrix}
\]

Or,

\[1 \cdot (a_{11} \cdots a_{1n}) + 1 \cdot (m_{11} \cdots m_{1n}) + (\nu_1 \cdots \nu_n) \equiv (1 \cdots 1)\]

Or,

\[uA_d + uA_m + A_z \equiv u\]
\[(4) \quad DVA^{\text{total}} = A_z (I - A_d)^{-1}\]
\[(5) \quad FVA^{\text{total}} = u A_m (I - A_d)^{-1} = u - A_z (I - A_d)^{-1}\]
WIOD and TiVA databases

The World Input-Output Database (WIOD):

World Input-Output Tables and underlying data, covering 43 countries, and a model for the rest of the world for the period 2000-2014. Data for 56 sectors are classified according to the International Standard Industrial Classification revision 4 (ISIC Rev. 4).

TiVA Database (OECD trade in Value Added Database):

The 2018 edition of the TiVA database provides indicators for 64 economies including all OECD, EU28 and G20 countries, most East and South-east Asian economies and a selection of South American countries.
Domestic value addition and Job creation by exports

1. DVA and FVA Contents of Export

The share of domestic value-added content of export ($DVA_{exp_i}^{total}$) to total export for $i^{th}$ industry can be obtained from:

$\forall i=n$ industries

\[
DVA_{exp_i}^{total} = A_Z(I - A_d)^{-1} \frac{E_d}{\sum_i E_d}
\]

Where,

$E_d$: $nx1$ vector of sectoral export

$\sum_i E_d$: country total export i.e., sum of exports across $n$ industries.

$\frac{E_d}{\sum_i E_d}$ is the $nx1$ vector of export share to total export.
In a more general way, total direct and indirect input contents of export \( (FVA_{exp_i}^{total}) \) is computed as:

\[
FVA_{exp_i}^{total} = uA_m(I - A_d)^{-1} \frac{E_d}{\Sigma_i E_d}
\]

\( \forall i = n \) industries

\( FVA_{exp_i}^{total} \) is the \( nxl \) vector.

2. Jobs embodied in net export

It can be calculated as: \( emp_i = LC \cdot (I - A_d)^{-1} \cdot TM \)

Where, \((I - A_d)^{-1} = \begin{pmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{n1} & \cdots & d_{nn} \end{pmatrix} \): inverse Leontief matrix. \( LC = \begin{pmatrix} l_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & l_n \end{pmatrix} \)

labour coefficient \((l_i)\) for a specific sector \(i\) is calculated as follows: \( employment_i/output_i \).
\[ TM = \begin{pmatrix} NE_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & NE_n \end{pmatrix} \]

\[ emp_i = LC \cdot (I - A_d)^{-1} \cdot TM \]

\[ = \begin{pmatrix} e_{11} & \cdots & e_{1n} \\ \vdots & \ddots & \vdots \\ e_{n1} & \cdots & e_{nn} \end{pmatrix} \begin{pmatrix} NE_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & NE_n \end{pmatrix} \]

\[ = \begin{pmatrix} e_{11} \cdot NE_1 & \cdots & e_{1n} \cdot NE_n \\ \vdots & \ddots & \vdots \\ e_{n1} \cdot NE_1 & \cdots & e_{nn} \cdot NE_n \end{pmatrix} \]

The sum of Row 1 represents the number of jobs created in Sector 1 through the net exports of all sectors. The sum of Column 1 represents the number of jobs created in all sectors of the economy, due to the net exports of Sector 1.
Calculations in MATLAB using WIOD database

MATLAB Codes to calculate DVA and FVA:

```matlab
i_d_data = S1(1:56,1:56);
y_d_data = S1(120,1:56);
i_m_data = S1(57:112,1:56);
A_d = bsxfun(@rdivide,i_d_data,y_d_data);
A_m = bsxfun(@rdivide,i_m_data,y_d_data);
A_d(isnan(A_d)) = 0;
A_m(isnan(A_m)) = 0;
I = eye(56,56);
iA_d = inv(I-A_d);
VA=bsxfun(@rdivide, S1(118,1:56),S1(120,1:56));
VA(isnan(VA)) = 0;
u = ones(1,56);
DVA = VA * iA_d;
FVA=1-DVA;
FVA_direct=u*A_m
```
### Literature Review:

**Import intensity of export of India**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Studies by</th>
<th>Database used</th>
<th>Measures</th>
<th>Period</th>
<th>Import intensity (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Import intensity of manufacturing</td>
<td>1979-80</td>
<td>7.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1973-74</td>
<td>10.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1979-80</td>
<td>8.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1979-80</td>
<td>11.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1983-84</td>
<td>12.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>for the whole economy</td>
<td>1979-80</td>
<td>4.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1983-84</td>
<td>5.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>listed in CMIE database</td>
<td>manufacturing sector</td>
<td>1997-98</td>
<td>12.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1998-99</td>
<td>12.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Import intensity for the manufacturing sector</td>
<td>1993-94</td>
<td>12.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1998-99</td>
<td>16.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2006-07</td>
<td>17.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Firm-level analysis using data</td>
<td>Import intensity of exporting firms</td>
<td>1999-00</td>
<td>9.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>from Capital line</td>
<td></td>
<td>2010-11</td>
<td>13.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Import intensity of non-exporting firms</td>
<td>1999-00</td>
<td>5.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2010-11</td>
<td>5.46</td>
</tr>
<tr>
<td>7.</td>
<td>Golder et al. (2017)</td>
<td>ASI industry level data</td>
<td>Import content in Indian exports (including</td>
<td>1995</td>
<td>11.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>service trade</td>
<td>2011</td>
<td>22.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Import content in merchandise exports</td>
<td>1995</td>
<td>11.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2011</td>
<td>26.00</td>
</tr>
<tr>
<td></td>
<td>(present study)</td>
<td></td>
<td></td>
<td>2014</td>
<td>25.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Import content of output in manufacturing</td>
<td>2000</td>
<td>19.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2014</td>
<td>25.80</td>
</tr>
</tbody>
</table>
Domestic and Foreign Value-Added contents in Aggregate Manufacturing of India
DVA content of export has fallen by 1.61 percent per annum during 2000-2014.
FVA content of export has increased by 5.18 percent per annum during 2000-2014.
OECD estimates of DVA and FVA indicators of Select countries (in percentage)
# Relationship between SAM and the National Accounts

**SAM: Endogenous and Exogenous Accounts**

<table>
<thead>
<tr>
<th>ACCOUNT</th>
<th>Endogenous</th>
<th>Exogenous</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Commodity</td>
<td>Intermediate consumption</td>
<td>Household final consumption expenditures</td>
<td>Other final demands</td>
</tr>
<tr>
<td>Activities</td>
<td>Domestic supplies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factors</td>
<td>Value added</td>
<td>Factor income from abroad</td>
<td>Total factor income receipts</td>
</tr>
<tr>
<td>Households</td>
<td>Factor income to households</td>
<td>Inter-household transfers</td>
<td>Non-factor income receipts</td>
</tr>
<tr>
<td>Other accounts (Exogenous)</td>
<td>Imports, indirect taxes</td>
<td>Other factor payments</td>
<td>Total household incomes</td>
</tr>
<tr>
<td>TOTAL</td>
<td>Total supply of products</td>
<td>Total activity payments</td>
<td>Total factor income payments</td>
</tr>
</tbody>
</table>
Comparison of characteristics of I-O, SAM and CGE models

• For the purpose of carrying out CGE Analysis, one requires input-output (I-O) and/or social accounting matrix (SAM) tables.

• Being the core of the SAM, the I-O framework describes the flows of value of goods and services between all the individual sectors of the national economy over a certain period (e.g. usually for a year).

• The I-O table can be extended into a SAM by adding information explaining the relationship between production factors and final demands; which shows the amount of income distributed to households and government as well as being transferred abroad and invested.
Thank You

Questions?